

Comparison of Galerkin and Control Volume Finite Element Schemes for Advection-Diffusion Problems

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The intent of developers of the control volume finite element method (CVFEM) was to produce a locally conservative method while retaining the unstructured grid and generality of finite element methods (FEMs). Similarly, the streamline upwind control volume (SUCV) method is a control volume version of the streamline upwind Petrov-Galerkin (SUPG) method designed to treat advectively dominated flows. These discretization methods are in current use and/or are candidates for new applications codes in the fluid and thermal sciences. In this talk we compare the CVFEM schemes with traditional FEM, paying particular attention to the conservation properties of the methods. We begin by reviewing the formulation of the CVFEM/SUCV in the context of a weighted residual method in which the weighting function is a distribution. Next we analyze its conservation properties, in particular showing that calculation of consistent fluxes on Dirichlet boundaries is necessary to assure global conservation. Finally, GFEM/SUPG and CVFEM/SUCV are compared via numerical error analysis in the context of advection-diffusion problems with respect to accuracy, conservation, degree of positivity, and convergence rate. For resolved, nonsingular problems we find no apparent advantage to the locally conservative formulation.